Does Boson Sampling need Fault-Tolerance?

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ABSTRACT

BosonSampling is a problem where a quantum computer offers a provable speedup over classical computers. Its main feature is that it can be solved with current linear optics technology, without the need for a full quantum computer.

We investigate whether an experimentally realistic BosonSampler can really solve BosonSampling without any fault-tolerance mechanism.

More precisely, we study how the unavoidable errors linked to an imperfect calibration of the optical elements affect the final result of the computation.

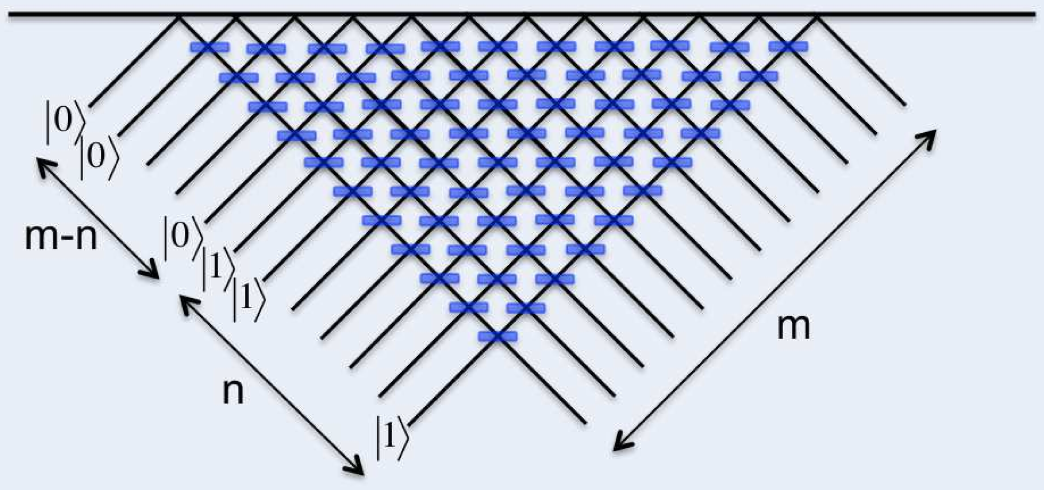
We show that the fidelity of each optical element must be at least 1 - O(1/n^2), where n refers to the number of single photons in the scheme. Such a requirement seems to be achievable with state-of-the-art equipment.

THE SETUP

BosonSampling(m, n):

INPUT: a unitary matrix U drawn from the Haar measure on U(m)

OUTPUT: a sample (s_1, …, s_m) from the distribution obtained by measuring the output modes of the following network in the photon number basis. More precisely, m input modes (n of which contain a single photon) are processed through an array of beamsplitters and phase-shifters that implement the unitary U (on the annihilation operators).



$$\Pr_U[s_1, s_2 \dots s_m] = \frac{\left|Per\left(U_{s_1 \cdots s_m}\right)\right|^2}{s_1! s_2! \cdots s_m!}$$

where s_i is the number of photons in the i^{th} output mode and $U_{s1,\dots,sm}$ is the n x n submatrix obtained by taking the first n rows of U and repeating the i^{th} column s_i times.

Aaronson and Arkhipov [1] showed that BosonSampling is hard for a classical computer, and provide evidence for hardness even in the approximate version of the problem, where we want to sample from a distribution ε-close to the ideal one in trace distance.

CAN A REALISTIC BOSONSAMPLER SOLVE THE PROBLEM?

A realistic experimental BosonSampler [2-5] will be subject to imperfections:

- losses [6]
- single photons not perfectly indistinguishable [7]
- imperfect detection efficiency [6]
- **optical network not implementing exactly the desired transformation [8]**

Reck et al [9] proposed a scheme to implement the unitary U as a network of beamsplitters and phase-shifters containing O(m^2) beamplitters.

Question: **how do errors made at each optical component propagate through the circuit?** And how do they affect the output distribution?

MODEL FOR NOISE

The noise affects each (complex) beamsplitter identically and independently:

- the beamplitter U_k acting on modes k_1 and k_2 is replaced by $\Phi(U_k)$ acting on the same two modes such that
- $\Phi(U_k)\,\Phi(U_k^\dagger) = \exp(i\varepsilon\, h_k)$ with h_k drawn from the Gaussian Unitary Ensemble (GUE)

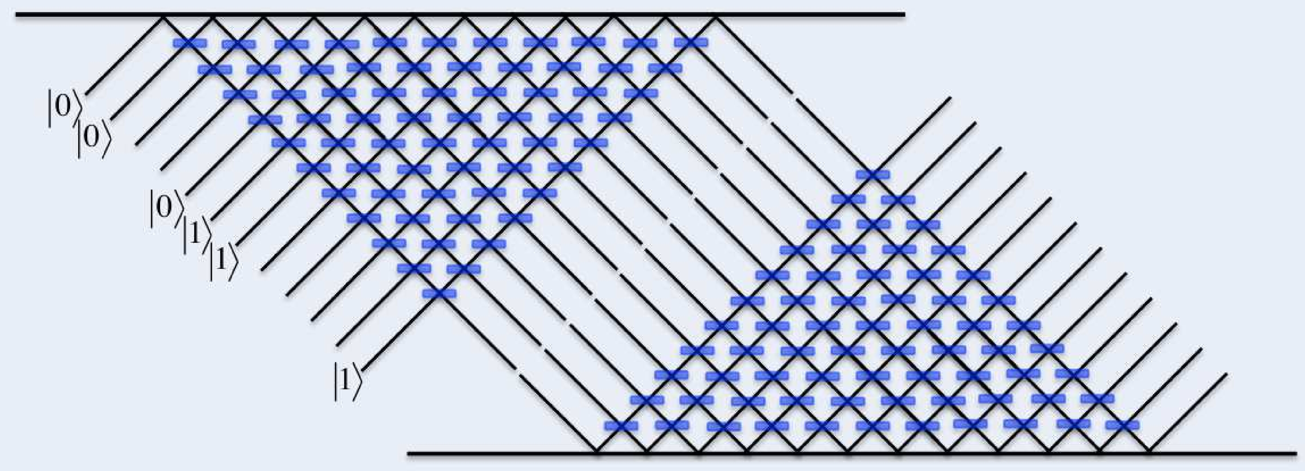
$$h_k = \begin{pmatrix} \alpha_k & \gamma_k \\ \gamma_k^* & \beta_k \end{pmatrix} \quad \text{with} \quad \alpha_k, \beta_k \sim N(0,1)_{\mathbb{R}}, \quad \gamma_k \sim N(0,1/2)_{\mathbb{C}}$$

Goal: value of $\mathrm{E}_U \mathrm{E}_\Phi \left\| \Pr_U - \Pr_{\Phi(U)} \right\|_1$

Unfortunately, very hard question, because of the permanent, and because the sum contains exponentially many terms, all exponentially small.

A MORE TRACTABLE PROBLEM

We consider the network U, concatenated with its inverse $U^\dagger$. Ideally, this network should implement the identity channel. We compute: $\mathrm{E}_U \mathrm{E}_\Phi \left\| \Pr_{Id} - \Pr_{\Phi(U^+)\Phi(U)} \right\|_1$



$$\mathrm{E}_U \mathrm{E}_\Phi \left\| \Pr_{Id} - \Pr_{\Phi(U^+)\Phi(U)} \right\|_1 = 2 - 2\mathrm{E}_U \mathrm{E}_\Phi \left| Per\left(\left[\Phi(U^+)\Phi(U) \right]_{n \times n} \right) \right|^2$$

where $[A]_{n \times n}$ is the nxn upper-left minor of A.

Main result: In the regime where $m=O(n^2)$, and $n^2\varepsilon^2 << 1$,

$$\mathrm{E}_U \mathrm{E}_\Phi \left\| \Pr_{Id} - \Pr_{\Phi(U^+)\Phi(U)} \right\|_1 = \Omega\left(n^2 \varepsilon^2\right).$$

Proof sketch:

- The matrix $\Phi(U^+)\Phi(U)$ can be written $\exp(i\varepsilon H_N)$ where H_N is the N^{th} step of a random walk on Hermitian matrices, and N is the number of beamsplitters.
- The Baker-Campbell-Hausdorf formula gives $H_N = \sum_{k=1}^{N} \Phi\left(U_n^\dagger\right) \cdots \Phi\left(U_k^\dagger\right) h_k \Phi\left(U_k\right) \cdots \Phi\left(U_n\right)$
- The permanent of the n×n upper-left minor of $\exp(i\varepsilon H_N)$ can be computed via $\mathrm{E}_x \left\| \left(I_m - \Pi_n \right) H_N \Pi_n x \right\|^2$ where $x = (e^{i\theta_1}, \dots, e^{i\theta_m})$ with θ_k uniform on $[0,2\pi]$ and $\Pi_n = \mathrm{diag}(1, \dots, 1, 0, \dots, 0)$.
- The expectations are handled with matrix concentration bounds due to Tropp [10].

CONCLUSION

The error does not scale too badly. For "reasonable" parameters, say m = 1000 modes, and n=30 single photons, the average fidelity per gate required to maintain the error on the order of a few percent is 0.999.

While such numbers certainly are challenging, they should not be a major problem in forthcoming BosonSampling experiments.

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